STUDY MATERIAL

TERM II

CLASS X

SUBJECT : MATHEMATICS

QUADRATIC EQUATIONS

KEY POINTS

1. The general form of a quadratic equation is $ax^2+bx+c=0$, $a\neq o$. a, b and c are real numbers.
2. A real number x is said to be a root of the quadratic equation $ax^2+bx+c=0$ where $a\neq o$ if $ax^2+bx+c=0$. The zeroes of the quadratic equation polynomial $ax^2+bx+c=0$ and the roots of the corresponding quadratic equation $ax^2+bx+c=0$ are the same.
3. Discriminant:- The expression b^2-4ac is called discriminant of the equation $ax^2+bx+c=0$ and is usually denoted by D. Thus discriminant $D= b^2-4ac$.
4. Every quadratic equation has two roots which may be real , co incident or imaginary.
5. IF α and β are the roots of the equation $ax^2+bx+c=0$ then

   $\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}$ And $\beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$

6. Sum of the roots , $\alpha + \beta = -\frac{b}{a}$ and product of the roots, $\alpha\beta = \frac{c}{a}$
7. Forming quadratic equation, when the roots α and β are given.
   $x^2-(\alpha+\beta)x+\alpha.\beta=0$
8. Nature of roots of $ax^2+bx+c=0$
   i. If $D>0$, then roots are real and unequal.
   ii. $D=0$, then the equation has equal and real roots.
   iii. $D<0$, then the equation has no real roots

LEVEL-I

1. IF ½ is a root of the equation $x^2+kx-5/4=0$, then the value of K is [Ans½]
2. IF $D>0$, then roots of a quadratic equation $ax^2+bx+c=0$ are [Ans$\frac{-b\pm\sqrt{D}}{2a}$]
3. Find the discriminant of $x^2+5x+5=0$. [Ans5]

4. The sum of roots of a quadratic equation x^2+4x-320=0 is

[Ans -4]

5. The product of roots of a quaradatic equation $2\,x^2$+7x-4=0 is

[Ans -2]

6. Values of K for which the equation $9\,x^2$+2kx-1=0 has real roots are:

[Ansk≥3 or K≤-3]

LEVEL-II

1. For what value of k, x=a is a solution of equation x^2-(a+b)x+k =0 ?

Ans. K=ab

2. Represent the situation in the form of quadratic equation:-

Rohan 's mother is 26 years older than him . the product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

Ans .x^2+32x -273 = 0 where x(in years) is Rohan's present age

3. Find the roots of x^2 -3x-10 = 0

Ans . -2 ,5

4. Find two consecutive positive integers , sum of whose squares is 365.

Ans .13,14

5. Find the roots of Quadratic equation $4x^2$ +4$\sqrt{3}x$+3 = 0 by using the quadratic formula.

Ans . - $\sqrt{\frac{3}{2}}$, - $\sqrt{\frac{3}{2}}$

6. Find the discriminant of the Quadratic equation $2x^2$-4x+3 = 0 and hence find the nature of its roots .

Ans . D= -8<0 its no real roots.

LEVEL - 3

1. If $x = 2\ and\ x = 3$ are roots of the equation $3x^2 - 2kx + 2m = 0$ find the value of k and m.

Ans.$K = \frac{15}{2}, m = 9$

2. Solve the equation:

$$\frac{x}{x+1}+\frac{x+1}{x}=\frac{34}{15}, x \neq 0, x \neq -1$$

Ans.$x=\frac{3}{2}\ or\ x=\frac{-5}{2}$

3. Solve the equation $2x^2-5x+3=0$ by the method of completing square.

Ans.$x=\frac{3}{2}\ or\ x=1$

4. Using quadratic formula, solve the equation: $p^2x^2+(p^2-q^2)x-q^2=0$.

Ans.$x=-1, or\ x=\frac{q^2}{p^2}$

5. The sum of two numbers is 15, if the sum of their reciprocals is $\frac{3}{10}, find\ the\ numbers.$

Ans. 10 and 5

[LEVEL - 4]

1. In a class test , the sum of shefali's marks in maths and English is 30. Had she got 2 marks more in maths and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Ans. Marks in maths = 12 , marks in English =18 or ,marks in maths = 13 , marks in English = 17

2. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Ans . 15 hours , 25 hours.

3. Find the roots of equation $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{13}$, x≠-4 , 7

Ans .1 , 2

4. Solve the following equation for 'x'
$9x^2$ - 9(a+b)x + ($2a^2$+5ab+$2b^2$) = 0

Ans .$\frac{2a+b}{3}$, $\frac{a+2b}{3}$

5. If the roots of the equation (a-b) x^2+(b-c)x + (c-a) = 0 are equal , prove that 2a =b+c.

Self Evaluation

1. Find the value of p so that the equation $3x^2 - 5x + 2p = 0$ has equal roots. Also find the roots.
2. The sum of two numbers is 15. If the sum of their reciprocals is $\frac{3}{10}, find\ the\ two\ numbers.$
3. Find a and b such that x+1 and x+2 are factors of the polynomials $x^3 + ax^2 - bx + 10.$
4. Find the quadratic equation whose roots are 2 + $\sqrt{3}$ and 2 - $\sqrt{3}$
5. A person on tour has Rs. 360 for his daily expenses. If he exceeds his tour program me by four days, he must cut down his daily expenses by Rs 3 per day. Find the number of days of his tour program me.
6. Divide 29 into two parts so that the sum of squares of the parts is 425.
7. Solve for x: $9x^2 - 6ax + (a^2 - b^2) = 0$
8. If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0\ has\ equal\ roots,$ show that $c^2 = a^2(1 + m^2)$

ARITHMETIC PROGRESSION

(Key Points)

- Arithmetic progression (A.P.) :- An A.P. is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.
- This fixed number is called the common difference of the A.P.
- If a is first term and d is common difference of an A.P. , then the A.P is a , a+d , a+2d , 2+3d …..
- The n^{th} term of an a.p is denoted by a_n anda_n = a+(n-1) d , where a = first term and d = common difference.
- n^{th}term from the end = l – (n-1) d , where l = last term.
- Three terms a-d , a , a+d are in A.P with common difference d.
- Four terms a-3d , a-d , a+d ,a+3d are in A.P with common diff. 2d .
- The sum of first n natural number is $\frac{n(n+1)}{2}$
- The sum of n terms of an A.P with first term a and common difference d is denoted by $s_n = \frac{n}{2}${ 2a+(n-1) d } also , $s_n = \frac{n}{2}$(a+l) where , l = last term.
- $a_n = s_n - s_{n-1}$. Where a_n=n^{th} term of an A.P
- D = $a_n - a_{n-1}$. Where d = common difference of an A.P.

[LEVEL -1]

1. Find n^{th} term of – 15 , -18 , -21 , ...

   Ans .-3 (n+4)

2. Find the common diff. of A.P 1 , -2 ,-5 ,-8 ,…….

   Ans . -3

3. Find the A.P whose first term is 4 and common difference is – 3

   Ans .a.p = 4 , 1 -2, -5, -8……

4. Find 5^{th} term from end of the AP : 17 , 14 ,11……-40.

   Ans . -28

5. If 2p, p+10 , 3p+2 are in AP then find p.

   Ans . p= 6

6. If arithmetic mean between 3a and 2a-7 is a+4 , then find a.

Ans . a= 5

7. Find sum of all odd numbers between 0 & 50.

Ans . 625

8. If a = 5 , d = 3 and a_n = 50 , then find n.

Ans .n =16

9. For what value of n are the n^{th} term of two AP , 63 , 65 , 67 ,…… and 3 , 10 , 17 ,…….equal?

Ans . n = 13.

10. If sum of n terms of an AP is $2n^2$+5n , then find its n^{th} term.

Ans. 4n+3.

[LEVEL - 2]

1. Find n^{th} term of an AP is 7-4n. find its common difference.

Ans. -4.

2. Which term of an AP 5,2,-1,….will be -22 ?

Ans .10^{th}term .

3. Write the next term of an AP$\sqrt{8}$,$\sqrt{18}$,$\sqrt{32}$,…….

Ans.$5\sqrt{2}$.

4. Determine 27^{th} term of an AP whose 9^{th} term is -10 and common difference is1 $\frac{1}{4}$

Ans. 927 = $\frac{25}{2}$.

5. Find the sum of series 103=+101+99+…..49.

Ans. 2128.

6. Which term of the AP 3,15,27,39,….will be 132 more than its 54^{th} term ?

Ans. 65^{th}term .

7. How many three digit numbers are divisible by 7 ?

Ans. 128.

8. Given a = 2 , d = 8 , s_n = 90 , find n and a_n.

Ans. N = 5 &a_n = 34

(LEVEL- 3)

1. Which term of the sequence -1, 3, 7, 11 ……. Is 95?

   Ans. 25th term
2. How many terms are there in the sequence 3, 6, 9, 12, ……111?

   Ans. 37 terms
3. The first term of an AP is -7 and the common difference 5, find its 18th term and the general term.

   Ans. a_{18} =78n & a_n = 5n – 12
4. How many numbers of two digits are divisible by 3?

   Ans. 30
5. If the nth term of an AP is (2n+1), find the sum of first n terms of the AP.

   Ans. S_n= n(n+2)
6. Find the sum of all natural numbers between 250 and 1000 which are exactly divisible by 3.

   Ans. 156375.

Problems for self evaluation.

1. Show that the sequence defined by t_n=4$_n$+7 is an AP.
2. Find the number of terms for given AP :7,13 ,19,25,…..,205.
3. The 7th term of an AP is 32 and it 13th term is 62. Find AP.
4. Find the sum of all two digit odd positive nos.
5. Find the value of ‘x’ for AP. 1+6+11+16+….+X=148.
6. Find the 10th term from the end of the AP 8,10,12,…126.
7. The sum of three numbers of AP is 3 and their product is -35.Find the numbers.
8. A man repays a loan of Rs3250 by paying Rs20 in the first month and then increase the payment by Rs15 every month .How long will it take him to clear the loan ?
9. The ratio of the sums of m and n terms of an AP is m^2 :n^2 .show that the ratio of the mth and nth terms is (2m-1) : (2n-1).

10. In an AP , the sum of first n terms is $\frac{3n^2}{2} + \frac{5n}{2}$, Find it 25^{th} term.

CO-ORDINATE GEOMETRY

IMPORTANT CONCEPTS
TAKE A LOOK

1. **Distance Formula**:-

The distance between two points $A(x_1,y_1)$ and $B(x_2,y_2)$ is given by the formula.

$AB=\sqrt{(X_2-X_1)^2+(Y_2-Y_1)^2}$

COROLLARY:- The distance of the point P(x,y) from the origin 0(0,0) is give by

$OP=\sqrt{(X-0)^2+(Y-0)^2}$ ie $OP=\sqrt{X^2+Y^2}$

2. Section Formula :-

the coordinates of the point $P(x, y)$ which divides the line segment joining points $A(x_1, y_1)$ and $B(x_2, y_2)$, internally, in the ratio $m_1 : m_2$ are

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \right)$$

3. Midpoint Formula:-

If R is the mid-point, then $m_1=m_2$ and the coordinates of R are

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$$

4. Co-ordinates of the centroid of triangle:-

The co-ordinates of the centroid of a triangle whose vertices are $P(x_1,y_1)$, $Q(x_2,y_2)$ and $R(x_3,y_3)$ are

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3} \right)$$

Area of a Triangle:-

The area of the triangle fromed a by the points $P(x_1,y_1)$ $Q(x_2,y_2)$ and $R(x_3,y_3)$ is the numerical value of the expression.

ar (ΔPQR)=1/2 $\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}$

LEVEL- 1

1. If the coordinates of the points P and Q are (4,-3) and (-1,7). Then find the abscissa of a point R on the line segment PQ such that $\frac{PR}{PQ} = \frac{3}{5}$

Ans. 1

2. If P $(\frac{a}{3},4)$ is the midpoint of the line segment joining the points Q (-6 , 5) and R (-2 , 3) , then find the value of a .

Ans . -12

3.A line intersects y –axis and x-axis at the points P and Q respectively . If (2 ,-5) is the midpoint of PQ , then find the coordinates of P and Q respectively .

Ans. (0,-10) and (4,0)

4. If the distance between the points (4,p)&(1,0) is 5,then find the value of p.

Ans.± 4

5. If the point A(1,2), B(0,0) and C(a,b)are collinear, then find the relation between a and b.

Ans. 2a=b

6. Find the coordinate of the point on x-axis which is equidistant from (2,-5)and (-2,9).

Ans. (-7,0)

7. Find the coordinates of a point A, where AB is diameter of a circle whose centre is (2, -3) and B is (1, 4)

Ans. (3, -10)

7. Find the centroid of triangle whose vertices are (3, -7), (-8, 6) and (5, 10).

Ans. (0, 3)

LEVEL-2

1. Point P (5, -3) is one of the two points of trisection of the line segment joining the points A (7, -2) and B (1, -5) near to A. Find the coordinates of the other point of trisection.

Ans. (3, -4)

2. Show that the point P (-4, 2) lies on the line segment joining the points
   A (-4 , 6) and B (-4, -6).
3. If A (-2, 4) ,B (0, 0) , C (4, 2) are the vertices of a ΔABC, then find the length of median through the vertex A.

Ans. 5 units

4. Find the value of x for which the distance between the points P (4, -5) and $Q(12, x)$ Is 10 units .

Ans. 1, -11

5. If the points A (4,3) and B (x,5) are on the circle with centre O(2,3) then find the value of x.

Ans. 2

6. What is the distance between the point A (c, 0) and B (0, -c)?

Ans. $\sqrt{2}$ c

7. For what value of p, are the points (-3, 9) , (2, p) and (4, -5) collinear?

Ans.$p = -1$

LEVEL-3

1. Show that the points (3, 2) , (0, 5) , (-3,2) and (0, -1) are the vertices of a square.
2. Point P divides the line segment joining the points A(2,1) and B(5,-8) such that AP:AB=1:3.If P lies on the line 2x-y+k=0,then find the value of k.

Ans. k = -8

3. Points P, Q , R, and S in that order are dividing a line segment joining A (2, 6) and B (7, -4) in five equal parts. Find the coordinates of point P and R ?

Ans. P (3, 4) , R (5, 0)

4. Find a relation between x and y if the points (2, 1) , (x, y) and (7, 5) are collinear.

Ans. 4x - 5y + 3 = 0

5. If A (-4, -2) , B (-3, -5) , C (3, -2) and D (2, 3) are the vertices of a quadrilateral, then find the area of the quadrilateral.

Ans. 28 sq. units

6. Find the values of x for which the distance between the points P(2, -3) and $Q(x, 5)$ is 10 units

Ans.$x = 8 \, or \, x = -4$

7. Find the point on y- axis which is equidistant from the points (5, -2) and (-3, 2)

Ans. (0, -2)

LEVEL-4

1. A (6, 1), B (8, 2), C (9, 4) are the three vertices of a parallelogram ABCD. If E is the midpoint of DC, then find the area of ΔADE .

Ans.$\frac{3}{4} sq. unit$

2. In each of following , find the value of 'k' for which the points are collinear .

(a) (7, -2) , (5, 1) , (3, k) (b) (8, 1) , (k, -4) ,(2,-5)

Ans.$(a)\, k = 4 \quad (b)\, k = 3$

3. Find the area of the triangle formed by joining the mid points of the sides of the triangle whose vertices are (0, -1) , (2,1) and (0,3). Find the ratio of this area to the area of the given triangle.

Ans. 1:4

4. Find the coordinates of the points which divides the line segment joining the points (-2,0) and (0,8) in four equal parts.

Ans. $(\frac{-3}{2}, 2), (-1,4), (-\frac{1}{2}, 6)$

5. Find the area of the quadrilateral whose vertices taken in order are (-4, -2), (-3, -5), (3, -2) and (2,3)

Ans. 28 sq. units

6. Find the area of the rhombus, if its vertices are (3,0), (4,5), (-1,4) and (-2,-1) taken in order.

Ans. 24 sq. units

HOTS /SELF EVALUATION

1. Two opposite vertices of a square are (-1,2) and (3, 2). Find the coordinates of the other two vertices.

   [Ans.(1,0) and (1,4)]
2. Find the centre of a circle passing through the points (6,-6), (3, 7) and (3,3).

   [Ans.3,-2]
3. If the distance between the points (3,0) and (0,y) is 5 units and y is positive, then what is the value of y? [Ans.4]
4. If the points (x,y) ,(-5,-2) and (3,-5) are collinear, then prove that $3x+8y+31 = 0$.
5. Find the ratio in which the Y-axis divides the line segment joining the points (5, -6) and (-1, -4). Also find the coordinates of the point of division.

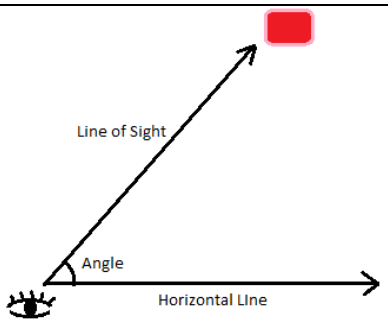
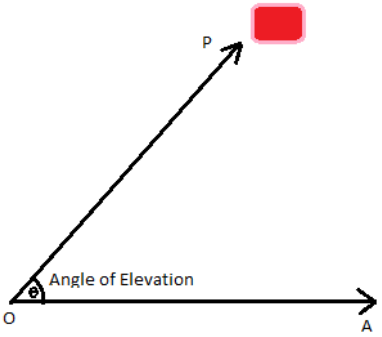
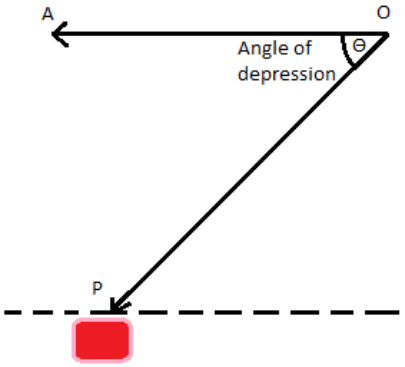
   Ans. 5:1; (0,-13/3)
6. Find k so that the point P(-4,6) lies on the line segment joining A (k,0) and B (3, -8). Also find the ratio in which P divides AB.

   [Ans. 3:7 externally; k=-1]
7. By distance formula, show that the points (1, -1), (5,2) and (9,5) are collinear.

APPLICATIONS OF TRIGONOMETRY

(HEIGHT AND DISTANCES)

KEY POINTS

Line of sight Line segment joining the object to the eye of the observer is called the line of sight.	
Angle of elevation When an observer sees an object situated in upward direction, the angle formed by line of sight with horizontal line is called angle of elevation.	
Angle of depression When an observer sees an object situated in downward direction the angle formed by line of sight with horizontal line is called angle of depression.	

LEVEL- 1

1. A $ploe$ 6cm high casts a shadow $2\sqrt{3}$m long on the ground, then find the sun's elevation?

   Ans. 60^0

2. If $\sqrt{3}tan\vartheta = 1, then\ find\ the\ value\ of\ sin^2\theta - cos^2\vartheta$

   Ans. -1/2

3. An observer 1.5m tall is 20.5 metres away from a tower 22m high. Determine the angle of elevation of the top of the tower from the eye of the observer.

Ans. 45°

4. A ladder 15m long just reaches the top of vertical wall. If the ladder makes an angle 60^0 with the wall, find the height of the wall

Ans. 15/2 m

5. In a rectangle ABCD, AB =20cm $\angle$BAC=60^0 then find the length of the side AD.

Ans. $20\sqrt{3}$cm

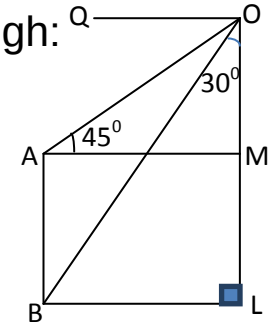
6. Find the angle of elevation of the sun's altitude when the height of the shadow of a vertical pole is equal to its height:

Ans. 45^0

7. From a point 20m away from the foot of a tower ,the angle of elevation of top of the tower is 30°,find the height of the tower.

Ans.$\frac{20}{\sqrt{3}}$m

8. In the adjacent figure, what are the angles of depression of the top and bottom of a pole from the top of a tower h m high:



Ans45^0, 60^0

LEVEL -2

1. In

$\Delta ABC, \angle B = 45^0,$

$\angle C = 45^0, A\,B = 5cm\ then\ find\ the\ length\ of\ the\ other\ two\ sides.$

Ans.$5cm, 5\sqrt{2}cm$

2. From a point 20 m away from the foot of the tower, the angle of elevation of the top of the tower is 30^0, find the height of the tower.

Ans.$\frac{20\sqrt{3}}{3}m$

3. A ladder 50m long just reaches the top of a vertical wall. If the ladder makes an angle of 60^0 with the wall, find the height of the wall.

Ans. 25 m

4. A circus artist is climbing a 20m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30^0.

Ans. 10 m

5. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30^0 with it. The distance between the foot of the tree to the point where the top touches the ground is 8m. Find the height of the tree.

Ans.$8\sqrt{3}m$

LEVEL - 3

1. The shadow of a tower standing on a level plane is found to be 50m longer when sun's elevation is 30^0 then when it is 60^0. Find the height of the tower.

Ans. $25\sqrt{3}m$

2. The angle of depression of the top and bottom of a tower as seen from the top of a 100m high cliff are 30^0 and 60^0 respectively. Find the height of the tower.

[Ans.66.67m]

3. From a window (9m above ground) of a house in a street, the angles of elevation and depression of the top and foot of another house on the opposite side of the street are 30^0 and 60^0 respectively. Find the height of the opposite house and width of the street.

[Ans.12m,$3\sqrt{3}$m]

4. From the top of a hill, the angle of depression of two consecutive kilometer stones due east are found to be 30^0 and 45^0. Find the height of the hill.

Ans.1.37 km

5. Two poles of equal heights are standing opposite each other on either side of the road ,which is 80m wide . From a point between them on the road the angles of elevation of the top of the poles are 60◦ and 30◦. Find the heights of pole and the distance of the point from the poles.

[Ans; h=34. 64m;

20m , 60m] .

6. The angle of elevation of a jet fighter from a point A on the ground is 60^0 .After a flight of 15 seconds, The angle of elevation changes to 30◦. If the jet is flying at a speed of 720km/ hr, find the constant height at which the jet is flying.

[Ans;1500m]

7. A window in a building is at a height of 10m above the ground . The angle of depression of a point P on the ground from the window is 30^0. The angle of elevation of the top of the building from the point P is 60^0 . Find the height of the building .

[Ans; 30m]

8. A boy , whose eye level is 1.3m from the ground , spots a ballon moving with the wind in a horizontal line at same height from the ground. The angle of elevation of the ballon from the eyes of the boy at any instant is 60^0. After 2 seconds , the angle of elevation

reduces to 30^0 If the speed of the wind at that moment is $29\sqrt{3}$ m/s , then find the height of the ballon from the ground .

[Ans; 88.3m]

9. A man on the deck on a ship 14m above water level , observes that the angle of elevation of the top of a cliff is $60^0$and the angle of depression of the base of the cliff is 30^0. Calculate the distance of the cliff from the ship and the height of the cliff . [Ans ; h= 56m , distance 24.25m]
10. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of tower with a uniform speed. Six minutes later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower.

[Ans.3 minutes]

SELF EVALUATION/HOTS

1. An aeroplane when flying at a height of 3125m from the ground passes vertically below another plane at an instant when the angle of elevation of the two planes from the same point on the ground are 30°and 60° respectively . Find the distance between the two planes at that instant .

[Ans ; 6250m]

2. From the top of a building 60m high , the angels of depression of the top and bottom of a vertical lamp post are observed to be 30° and 60°respectively. Find [i] horizontal distance between the building and the lamp post [ii] height of the lamp post .

[Ans. 34.64m h=40m]

3. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height h m. At a point on the plane, the angles of elevation of the bottom and the top of the flag staff are $\propto and\ \beta$, respectively. Prove that the height of the tower is $\frac{htan\propto}{tan\beta - tan\propto}$

4. The angle of elevation of a cloud from a point 60m above a lake is 30◦ and the angle of depression of the reflection of the cloud in the lake is 60°. Find the height of the cloud from the surface of the lake.

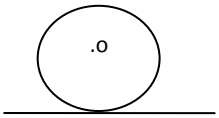
[Ans: 120m]

Circle

KEY POINTS

Tangent to a circle :

A tangent to a circle is a line that intersects the circle at only one point.



P tangent

P= point of contact

- There is only one tangent at a point on a circle.
- There are exactly two tangents to a circle through appoint lying outside the circle.
- The tangent at any point of a circle is perpendicularto the radius through the point of contact.
- The length of tangents drown from an external point to a circle are equal.

(1 Mark Questions)

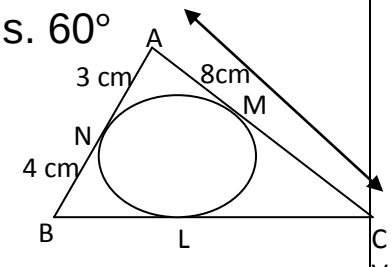
1. If radii of the two concentric circles are 15cm and 17cm, then find the length of each chord of one circle which is tangent to one other.

Ans. 16cm

2. If two tangents making an angle of 120^0 with each other are drawn to a circle of radius 6cm, then find the angle between the two radii, which are drawn to the tangents.

Ans. 60°

3. In the adjoining figure, Δ ABC is circumscribing a circle.Find the length of BC.



Ans. 9cm

4. PQ is a chord of a circle and R is point on the minor arc. If PT is a tangent at point P such that ∠QPT = 60° then find <PRQ.

Ans. 120°

5. If a tangent PQ at a point P of a circle of radius 5cm meets a line through the centre O at a point Q such that OQ = 12 cm then find the length of PQ.

Ans. $\sqrt{119}$cm

6. From a point P, two tangents PA and PB are drawn to a circle C(O,r) . If OP =2r, then what is the type of Δ APB.

Ans. Equilateral triangle

7. If the angle between two radii of a circle is 130°, then find the angle between the tangents at the end of the radii.

Ans. 50°.

8. ABCD is a quadrilateral. A circle with centreat O is inscribed in the quadrilateral. If AB = 7cm, BC = 4cm , CD = 5cm then find DA.

Ans. 8 cm

9. In aΔ ABC, AB = 8cm,∠ABC = 90°. Then find the radius of the circle inscribed in the triangle.

Ans. 2cm

(Two Marks Questions)

1. Two tangents PA and PB are drawn from an external point P to a circle with centre O. Prove that OAPB is a cyclic quadrilateral.

2. If PA and PB are two tangents drawn to a circle with centre O, from an external point P such that PA=5cm and ∠APB = 60°, then find the length of the chord AB.

Ans. 5cm

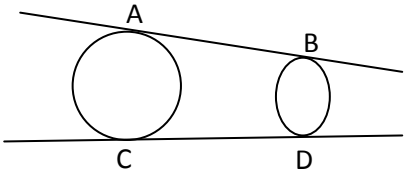
3. CP and CQ are tangents from an external point C to a circle with centre O .AB is another tangent which touches the circle at R and intersects PC and QC at A and B respectively . If CP = 11cm and BR = 4cm, then find the length of BC.

Ans: 7 cm.

4. If all the sides of a parallelogram touch a circle, show that the parallelogram is a rhombus.

5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle.

6. In adjacent figure; AB & CD are common tangents to two circles of unequal radii. Prove that AB=CD.



(Three Marks Questions)

1. If quadrilateral ABCD is drawn to circumscribe a circle then prove that AB+CD=AD+BC.
2. Prove that the angle between the two tangents to a circle drawn from an external point is supplementary to the angle subtended by the line segment joining the points of contact to the centre.
3. AB is a chord of length 9.6cm of a circle with centre O and radius 6cm.If the tangents at A and B intersect at point P then find the length PA.

Ans. 8cm

4. The incircle of a ΔABC touches the sides BC, CA &AB at D,E and F respectively. If AB=AC, prove that BD=CD.
5. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre of the circle.

6. PQ and PR are two tangents drawn to a circle with centre O from an external point P. Prove that $\angle QPR=2\angle OQR$.

(Four Marks Questions)

1. Prove that the lengths of tangents drawn from an external point to a circle are equal. Hence, find BC, if a circle is inscribed in a ΔABC touching AB,BC &CA at P,Q &R respectively, having AB=10cm, AR=7cm &RC=5cm.

   Ans. 8cm
2. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. Using the above, do the following: If O is the centre of two concentric circles, AB is a chord of the larger circle touching the smaller circle at C, then prove that AC=BC.
3. A circle touches the side BC of a ΔABC at a point P and touches AB and AC when produced, at Q & R respectively. Show that AQ=1/2 (perimeter of ΔABC).
4. From an external point P, a tangent PT and a line segment PAB is drawn to circle with centre O, ON is perpendicular to the chord AB. Prove that $PA.PB=PN^2-AN^2$.
5. If AB is a chord of a circle with centre O, AOC is diameter and AT is the tangent at the point A, then prove that $\angle BAT=\angle ACB$.
6. The tangent at a point C of a circle and diameter AB when extended intersect at P. If $\angle PCA=110^0$, find $\angle CBA$.

   Ans. 70^0

[Self Evaluation/HOTS Questions]

1. If PA and PB are tangents from an external point P to the circle with centre O, the find $\angle AOP+\angle OPA$.

   Ans. 90^0

2. ABC is an isosceles triangle with AB=AC, circumscribed about a circle. Prove that the base is bisected by the point of contact.
3. AB is diameter of a circle with centre O. If PA is tangent from an external point P to the circle with $\angle POB=115^0$ then find $\angle OPA$.

   Ans. 25^0

4. PQ and PR are tangents from an external point P to a circle with centre. If $\angle RPQ=120^0$, Prove that OP=2PQ.
5. If the common tangents AB and CD to two circles C(O,r) and C'(O'r') intersect at E, then prove that AB=CD.
6. If a, b, c are the sides of a right triangle where c is the hypotenuse , then prove that radius r of the circle touches the sides of the triangle is given by r= (a+b-c)/2.

CONSTRUCTIONS

KEY POINTS

1. Division of line segment in the given ratio.

2. Construction of triangles:-

   a. When three sides are given.
   b. When two sides and included angle given.
   c. When two angles and one side given.
   d. Construction of right angled triangle.

3. Construction of triangle similar to given similar to given triangle as per given scale.

4. Construction of triangles to a circle.

LEVEL - I

1. Divide a line segment in given ratio.

2. Draw a line segment AB=8cm and divide it in the ratio 4:3.

3. Divide a line segment of 7cm internally in the ratio 2:3.

4. Draw a circle of radius 4 cm. Take a point P on it. Draw tangent to the given circle at P.

5. Construct an isosceles triangle whose base 7.5 cm and altitude is 4.2 cm.

LEVEL –II

1. Construct a triangle of sides 4cm, 5cm and 6cm and then triangle similar to it whose sides are 2/3 of corresponding sides of the first triangle.

2. Construct a triangle similar to a given ΔABC such that each of its sides is $2/3^{rd}$ of the corresponding sides of ΔABC. It is given that AB=4cm BC=5cm and AC=6cm also write the steps of construction.

3. Draw a right triangle ABC in which $\angle B=90^0$ AB=5cm, BC=4cm then construct another triangle ABC whose sides are 5/3 times the corresponding sides of ΔABC.

4. Draw a pair of tangents to a circle of radius 5cm which are inclined to each other at an angle of 60^0.

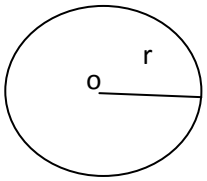
5. Draw a circle of radius 5cm from a point 8cm away from its centre construct the pair of tangents to the circle and measure their length.

6. Construct a triangle PQR in which QR=6cm $\angle Q=60^0$ and $\angle R=45^0$. Construct another triangle similar to ΔPQR such that its sides are 5/6 of the corresponding sides of ΔPQR.

AREAS RELATED TO CIRCLES

KEY POINTS

1. Circle: The set of points which are at a constant distance of r units from a fixed point o is called a circle with centre o.



2. Circumference: The perimeter of a circle is called its circumference.

3. Secant: A line which intersects a circle at two points is called secant of the circle.

4. Arc: A continuous piece of circle is called and arc of the circle..

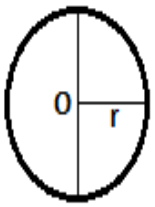
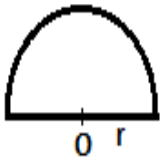
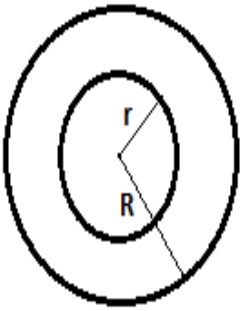
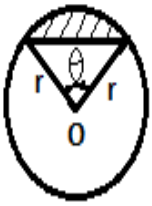
5. Central angle:- An angle subtended by an arc at the center of a circle is called its central angle.

6. Semi Circle: - A diameter divides a circle into two equal arc. Each of these two arcs is called a semi circle.

7. Segment :- A segment of a circle is the region bounded by an arc and a chord, including the arc and the chord.

8. Sector of a circle: The region enclosed by and an arc of a circle and its two bounding radii is called a sector of the circle.

9. Quadrant:- One fourth of a circle disc is called a quadrant. The central angle of a quadrant is 90^0.

S.N	NAME	FIGURE	PERIMETER	AREA
1.	Circle		$2\pi r$ or πd	πr^2
2.	Semi- circle		πr + 2r	½ πr^2
3.	Ring (Shaded region)		2 $\pi(r$ + R)	π(R^2-r^2)
4.	Sector of a circle		l+2r=$\frac{\pi r\theta}{180^\circ} + 2r$	$\frac{\pi r^2\theta}{360^\circ}$ or $\frac{1}{2}lr$
5.	Segment of a circle		$\frac{\pi r\theta}{180^\circ}$+2r Sin$\frac{\theta}{2}$	$\frac{\pi r^2\theta}{360^\circ}$- $\frac{1}{2}r^2$sinθ

a. Length of an arc AB= $\frac{\theta}{360}$ 2 πr

b. Area of major segment= Area of a circle – Area of minor segment

c. Distance moved by a wheel in
1 rotation=circumference of the wheel

d. Number of rotation in 1 minute

=Distance moved in 1 minute / circumference

LEVEL-I

1. If the perimeter of a circle is equal to that of square, then the ratio of their areas is
   i. 22/7
   ii. 14/11
   iii. 7/22
   iv. 11/14 [Ans-ii]

2. The area of the square that can be inscribed in a circle of 8 cm is
   i. 256 cm^2
   ii. 128 cm^2
   iii. $64\sqrt{2}$ cm^2
   iv. 64 cm^2 [Ans-ii]

3. Area of a sector to circle of radius 36 cm is 54 πcm^2 . Find the length arc of the corresponding arc of the circle is
   i. 6 πcm
   ii. 3 πcm
   iii. 5 πcm
   iv. 8 πcm

   [Ans –ii]

4. A wheel has diameter 84 cm. The number of complete revolution it will take to cover 792 m is.

i. 100
ii. 150
iii. 200
iv. 300 [Ans-iv]

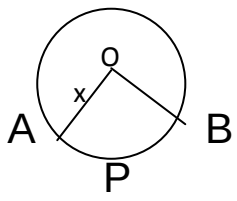
5. The length of an arc of a circle with radius 12cm is 10 π cm. The central angle of this arc is .
   i. 120^0 [Ans-iv]
   ii. 60^0
   iii. 75^0
   iv. 150^0

6. The area of a quadrant of a circle whose circumference is 22 cm is
   i. 7/2 cm^2
   ii. 7 cm^2
   iii. 3 cm^2
   iv. 9.625 cm^2 [Ans-iv]

LEVEL-II

1. In figure ‘o’ is the centre of a circle. The area of sector OAPB is 5/18 of the area of the circle find x.

[Ans 100]



2. If the diameter of a semicircular protractor is 14 cm, then find its perimeter. [Ans-36 cm]

3. The radii of two circles are 3 cm and 4 cm. Find the radius of a circle whose area is equal to the sum of the areas of the two circles.

[Ans: 5 cm]

4. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

[Ans: 154/3 cm]

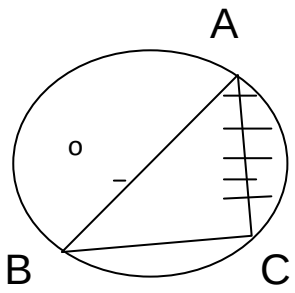
5. The radii of two circles are 3 cm and 4 cm. Find the radius of a circle whose area is equal to the sum of the areas of the two circles.

[Ans 5cm]

LEVEL-III

1. Find the area of the shaded region in the figure if AC=24 cm ,BC=10 cm and o is the center of the circle (use $\pi = 3.14$)

[Ans- 145.3cm^2]

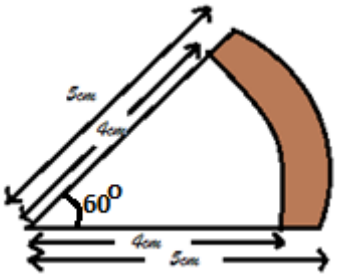


2. The inner circumference of a circular track is 440m. The track is 14m wide. Find the diameter of the outer circle of the track. [Take π =22/7]

[Ans-168]

3. Find the area of the shaded region.

[Ans: 4.71cm^2]



4. A copper wire when bent in the form of a square encloses an area of 121 cm^2. If the same wire is bent into the form of a circle, find the area of the circle (Use π=22/7)

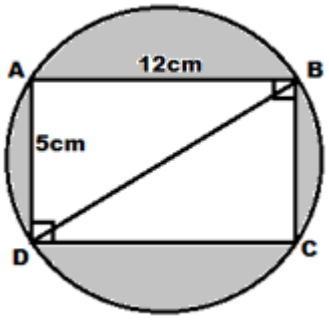
[Ans 154 cm^2]

5. A wire is looped in the form of a circle of radius 28cm. It is rebent into a square form. Determine the side of the square (use $\pi = 22/7$)

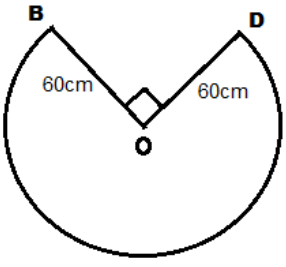
[Ans-44cm]

LEVEL-IV

1. In fig, find the area of the shaded region [use $\pi = 3.44$]



2. In fig find the shape of the top of a table in restaurant is that of a sector a circle with centre 0 and $\angle$bod=90^0. If OB=OD=60cm fnd:
   i. The area of the top of the table [Ans 8478 cm^2]
   ii. The perimeter of the table top (Take $\pi = 3.44$) [Ans 402.60 cm]



3. An arc subtends an angle of 90^0 at the centre of the circle of radius 14 cm. Write the area of minor sector thus form in terms of π.
[Ans 49π cm^2]

4. The length of a minor arc is 2/9 of the circumference of the circle. Write the measure of the angle subtended by the arc at the center of the circle.
[Ans 80^0]

5. The area of an equilateral triangle is $49\sqrt{3}$ cm^2. Taking each angular point as center, circles are drawn with radius equal to half the length of the side of the triangle. Find the area of triangle not included in the circles.
[Take $\pi\sqrt{3}=1.73$] [Ans $777cm^2$]

SELF EVALUATION

1. Two circles touch externally the sum of the areas is $130\ \pi$ cm^2 and distance between there center is 14 cm. Find the radius of circle.

2. Two circle touch internally. The sum of their areas is $116\ \pi$ cm^2 and the distance between there centers is 6 cm. Find the radius of circles.

3. A pendulum swings through an angle of 30^0 and describes and arc 8.8 cm in length. Find length of pendulum.

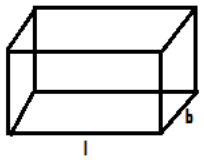
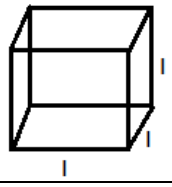
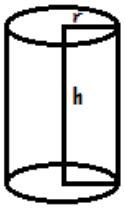
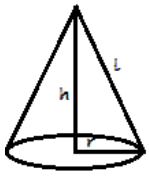
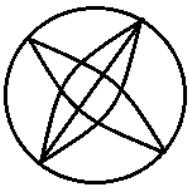
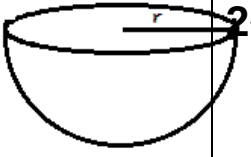
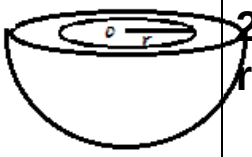
4. What is the measure of the central angle of a circle?

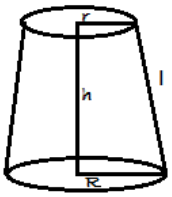
5. The perimeter and area of a square are numerically equal. Find the area of the square.

SURFACE AREAS AND VOLUMES

IMPORTANT FORMULA

TAKE A LOOK

SNo	NAME	FIGURE	LATERAL CURVED SURFACE AREA	TOTAL SURFACE AREA	VOLUME	NOMENCLATURE
1	**Cuboid**		**2(l+b)x h**	**2(lxb + bxh + hx l)**	**l x b x h**	**L=length, b=breadth, h=height**
2	**Cube**		**$4l^2$**	**$6l^2$**	**l^3**	**l=edge of cube**
3	**Right Circular Cylinder**		**$2\pi rh$**	**$2\pi r(r+h)$**	**$\pi r^2 h$**	**r= radius h=height**
4	**Right Circular Cone**		**πrl**	**$\pi r(l+r)$**	**$\frac{1}{3}\pi r^2 h$**	**r=radius of base, h=height , l=slant height = $\sqrt{r^2 - h^2}$**
5	**Sphere**		**$4\pi r^2$**	**$4\pi r^2$**	**$\frac{4}{3}\pi r^3$**	**r=radius of the sphere**
6	**Hemisphere**		**$2\pi r^2$**	**$3\pi r^2$**	**$\frac{2}{3}\pi r^3$**	**r=radius of hemisphere**
7	**Spherical shell**		**$2\pi(R^2 + r^2)$**	**$3\pi(R^2 - \pi r^2)$**	**$\frac{4}{3}\pi(R^3 - r^3)$**	**R=External radius, r=internal radius**

8	**Frustum of a cone**		**$\pi l(R+r)$ where $l^2=h^2+(R-r)^2$**	**$\pi[R^2 + r^2 + l(R+r)]$**	**$\pi h/3[R^2 + r^2 + Rr]$**	**R and r = radii of the base, h=height, l=slant height.**

9. Diagonal of cuboid = $\sqrt{l^2 + b^2 + h^2}$
10. Diagonal of Cube = $\sqrt{3}$ side

(LEVEL - 1)

[1] The height of a cone is 60 cm.A small cone is cut off at the top by a plane parallel to the base and its volume is $\frac{1}{64}^{th}$ the volume of original cone. Find the height from the base at which the section is made?

ANS :- 45 cm

[2] Find the volume of the largest right circular cone that can be cut out from a cube of edge 4.2 cm?

ANS:- 19.4 cm^3.

[3] A cubical ice cream brick of edge 22cm is to be distributed among some children by filling ice cream cones of radius 2cm and height 7cm up to its brim. How many children will get ice cream cones?

ANS :-363.

[4] Find the volume of the largest right circular cone that can be cut out from a cube of edge 4.9 cm is?

ANS :- 30.8cm^3.

[5] The slant height of a frustum of a cone is 4 cm and the perimeter of its circular ends are18cm and 6cm. Find the curved surface area of the frustum [use $\pi = \frac{22}{7}$].

ANS :- 48cm^2

[6] A plumb line is a combination of which geometric shapes?

ANS :-A cone with hemisphere.

LEVEL - 2

[1] The slant height of the frustum of a cone is 5 cm. If the difference between the radii of its two circular ends is 4cm,find the height of the frustum.

ANS :- 3cm

[2] A cylinder, a cone and a hemisphere are of same base and of same height .Find the ratio of their volumes?

ANS :- [3:1:2].

[3] A cone of radius 4cm is divided into two parts by drawing a plane through the midpoint of its axis and parallel to its base, compare the volume of the two parts.

ANS :- 1:7

[4] How many spherical lead shots each having diameter 3cm can be made from a cuboidal lead solid of dimensions 9cm X 11cm X 12cm.

ANS :- 84

[5] Three metallic solid cubes whose edges are 3cm, 4cm, and 5cm are melted and converted into a single cube .Find the edge of the cube so formed?

ANS :- 6cm .

(LEVEL-3)

[1] How many shots each having diameter 4.2 cm can be made from a cuboidal lead solid of dimensions 66cm X 42cm X 21cm?

ANS:-1500

[2] Find the number of metallic circular disk with 1.5cm base diameter and of height 0.2 cm to be melted to form a right circular cylinder of height 10cm and diameter 4.5cm?

ANS:-450

[3] From a solid cube of side 7cm, a conical cavity of height 7cm and radius 3cm is hollowed out. Find the volume of remaining solid?

ANS:-277cm^3.

[4] A cubical block of side 7cm is surmounted by a hemisphere. What is the greatest diameter of the hemisphere can have? Find the surface area of the solid?

ANS:- 7cm,332.5cm^2.

[5] A heap of rice is in the form of a cone of diameter 9m and height 3.5m .Find the volume of the rice .How much canvas cloth is required to just cover the heap?

ANS:-74.25m^3, 80.61 m^2 .

[6] A square field and an equilateral triangle park have equal perimeter .If the cost of ploughing the field at the rate of Rs 5/m^2 is Rs 720. Find the cost of maintain the park at the rate of Rs10/m^2?

ANS:-Rs1108.48

(LEVEL -4)

[1] A well of diameter 3cm and 14m deep in dug. The earth, taken out of it, has been evenly spread all around it in the shape of a circular ring of width 4m to form an embankment. Find the height of embankment?

ANS:-$\frac{9}{8}$ m.

[2] 21 glass spheres each of radius 2cm are packed in a cuboidal box of internal dimensions16cmX8cmX8cmand then the box is filled with water. Find the volume of water filled in the box?

ANS:-320cm^3.

[3] The slant height of the frustum of a cone is 4cm and the circumferences of its circular ends are 18cm and 6cm. Find curved surface area and total surface area of the frustum.

ANS:-48cm^2, 76.63cm^2.

[4] A farmer connects a pipe of internal diameter 25cm from a canal into a cylindrical tank in his field, which is 12m in diameter and 2.5m deep. If water flows through the pipe at the rate of 3.6km/hr, in how much time will the tank be filled? Also find the cost of water, if the canal department charges at the rate of Rs0.07/m^3?

ANS:-96min, Rs19.80

[5] A spherical glass vessel has a cylindrical neck 7cm long and 4cm in diameter. The diameter of the spherical part is 21cm. Find the quantity of water it can hold.

ANS:-4939cm^3.

[6] The surface area of a solid metallic sphere is 616cm^2. It is melted and recast into a cone of height 28cm. Find the diameter of the base of the cone so formed.

ANS:-14cm.

SELF EVALUTION/HOTS QUESTIONS

[1] A spherical copper shell , of external diameter 18cm,is melted and recast into a solid cone of base radius 14cm and height 4cm. Find the inner diameter of the shell.

ANS:-16cm.

[2] A bucket is in the form of a frustum of a cone with a capacity of 12308.8cm^3. The radii of the top and bottom circular ends of the bucket are 20cm and 12cm respectively. Findthe height of the bucket and also the area of metal sheet used in making it [take π=3.14]?

ANS:-$l = 14cm\,, AREA = 2160.32cm2$.

[3] The volume of a solid metallic sphere is 616cm^3.its is melted and recast into a cone of height 28cm. Find the diameter of the base of the cone so formed?

ANS:-21cm.

[4] From a solid cylinder whose height is 8cm and radius 6cm, a conical cavity of height 8cm and of base radius 6cm, is hollowed out. Find the volume of the remaining solid correct to two places of decimals. Also find the total surface area of the remaining solid [take π=3.14] ?

ANS:-603.19cm^3, 603.19cm^2 .

[5] A cylindrical vessel, with internal diameter10cm and height 10.5 cm is full of water. A solid cone of base diameter 7cm and height 6cm is completely immersed in water. Find the volume of:-

(i) water displaced out of the cylindrical vessel.

(ii) water left in the cylindrical vessel.

ANS:- (i): 77cm^3 , (ii) 748cm^3.

[6] A wooden article was made by scooping out a hemisphere from each ends of a solid cylinder. If the height of the cylinder is 20cm, and radius of the base is 3.5cm, find the total surface area of the article.

ANS:-544cm^2.

[7] A building is in the form of a cylinder surmounted by a hemispherical vaulted dome and contains $41\frac{19}{21}$ $m^3$of air. If the internal diameter of the building is equal to its total height above the floor, find the height of the building?

ANS:-4m .

[8] A shuttle cock used for playing badminton has the shape of a frustum of a cone mounted on a hemisphere. The external diameters of the frustum are 5cm and 2cm, the height of the entire shuttle cock is 7cm. Find the external surface area.

ANS:-74.38cm^2.

PROBABLITY

KEY POINTS

1. **Probability:**- The theoretical probability of an event E, written as P(E) is defined as.

P(E)= Number of outcomes Favorable to E
Number of all possible outcomes of the experiment. Where we assume that the outcomes of the experiment are equally likely.

2. The probability of a sure event (or certain event) is 1.

3. The probability of an impossible event is 0.

4. The probability of an Event E is number P (E) such that $0 \leq P(E) \leq 1$.

5. Elementary events:- An event having only one outcome is called an elementary event. The sum of the probabilities of all the elementary events of an experiment is 1.

6. For any event E,$P(E)+P(\bar{E})=1$, where$\bar{E}$ stands for not E, E and $\bar{E}$ are called complementary event.

7. Performing experiments:-
   a. Tossing a coin.
   b. Throwing a die.
   c. Drawing a card from deck of 52 cards.

8. **Sample space:-**The set of all possible outcomes in an experiment is called sample space.

LEVEL-1

1. The probability of getting bad egg in a lot of 400 is 0.035.Then find the no. of bad eggs in the lot. [ans.14]

2. Write the probability of a sure event.

[ans.1]

3. What is the probability of an impossible event. [ans.0]

4. When a dice is thrown, then find the probability of getting an odd number less than 3. [ans. $\frac{1}{6}$]

5. A girl calculates that the probability of her winning the third prize in a lottery is 0.08.If 6000 tickets are sold, how many ticket has she brought.

[Ans.480]

6. What is probability that a non-leap year selected at random will contain 53 Sundays. [Ans.$\frac{1}{7}$]

7. A bag contains 40 balls out of which some are red, some are blue and remaining are black. If the probability of drawing a red ball is $\frac{11}{20}$ and that of black ball is $\frac{1}{5}$, then what is the no. of black ball. [Ans.10]

8. Two coins are tossed simultaneously. Find the probability of getting exactly one head. [Ans.$\frac{1}{2}$]

9. A card is drawn from a well suffled deck of 52 cards. Find the probability of getting an ace.

[Ans.$\frac{1}{13}$]

10. In a lottery, there are 10 prizes and 25 blanks. Find the probability of getting a prize. [Ans.$\frac{2}{7}$]

LEVEL-2

1. Find the probability that a no. selected at random from the number 3,4,5,6,……..25 is prime. [Ans.$\frac{8}{23}$]

2. A bag contains 5 red,4 blue and 3 green balls. A ball is taken out of the bag at random. Find the probability that the selected ball is (a) of red colour (b) not of green colour.

[Ans.$\frac{5}{12}$,$\frac{3}{4}$]

3. A card is drawn at random from a well-shuffled deck of playing cards. Find the probability of drawing

(a) A face card (b)card which is neither a king nor a red card

[Ans.$\frac{3}{13}$,$\frac{6}{13}$]

4. A dice is thrown once. What is the probability of getting a number greater than 4? [Ans.$\frac{1}{3}$]

5. Two dice are thrown at the same time. Find the probability that the sum of two numbers appearing on the top of the dice is more than 9.

[Ans.$\frac{1}{6}$]

6. Two dice are thrown at the same time. Find the probability of getting different numbers on both dice. [Ans.$\frac{5}{6}$]

7. A coin is tossed two times. Find the probability of getting almost one head.

[Ans.$\frac{3}{4}$]

8. Cards with numbers 2 to 101 are placed in a box. A card selected at random from the box. Find the probability that the card which is selected

has a number which is a perfect square.

[Ans.$\frac{9}{100}$]

9. Find the probability of getting the letter M in the word "MATHEMATICS".

[Ans.$\frac{2}{11}$]

LEVEL-3

1. Cards bearing numbers 3,5,……..,35 are kept in a bag. A card is drawn at random from the bag.Find the probability of getting a card bearing (a)a prime number less than 15 (b)a number divisible by 3 and 5.

[Ans.$\frac{5}{17},\frac{2}{17}$]

2. Two dice are thrown at the same time. Find the probability of getting (a)same no. on the both side (b)different no. on both sides.

[Ans.$\frac{1}{6},\frac{5}{6}$]

3. A child game has 8 triangles of which three are blue and rest are red and ten squares of which six are blue and rest are red. One piece is lost at random. Find

the probability of that is (a) A square (b) A triangle of red colour.

4. Two dice are thrown simultaneously. What is the probability that:
(a)5 will not come up either of them? (b)5 will come up on at least one? (C)5 will come at both dice?

[Ans.$\frac{25}{36},\frac{11}{36},\frac{1}{36}$]

5. The king, queen and jack of clubs are removed from a deck of 52 playing cards and remaining cards are suffled. A card is drawn from the

remaining cards. Find the probability of getting a card of (a)heart (b)queen (c)clubs

[Ans.$\frac{13}{49}, \frac{3}{49}, \frac{10}{49}$]

6. A game consists of tossing a one-rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result, i.e., 3 heads or three tails and looses otherwise. Calculate the probability that Hanif will lose the game.

[Ans.$\frac{3}{4}$]

7. Cards bearing numbers 1,3,5,……..,37 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card bearing

(a)a prime number less than 15

[Ans. $\frac{5}{19}$]

(b)a number divisible by 3 and 5.

[Ans.$\frac{2}{19}$]

8. A dice has its six faces marked 0,1,1,1,6,6.Two such dice are thrown together and total score is recorded.(a)how many different scores are possible? (b)what is the probability of getting a total of seven?

[Ans.{a} 5 scores(0,1,2,6,7,12) {b }$\frac{1}{3}$]

Self Evaluation/Hots

1. Three unbiased coins are tossed together. find the probability of getting
   (i) all heads

Ans.$\frac{1}{8}$

(ii) two heads

Ans.$\frac{3}{8}$

(iii) one heads

Ans.$\frac{3}{8}$

(iv) at least two heads

Ans.$\frac{1}{2}$

2. Two dice are thrown simultaneously .Find the probability of getting an even number as the sum.

Ans.$\frac{1}{2}$

3. Cards marked with the number 2 to 101 are placed in a box and mixed thoroughly. One card is drawn from the box. Find the probability that the number on the card is:

(i) An even number

Ans. $\frac{1}{2}$

(ii) A number less than 14

Ans. $\frac{3}{25}$

(iii) A number is perfect square

Ans. $\frac{9}{100}$

(iv) A prime number less than 20

Ans. $\frac{2}{25}$

4. Out of the families having three children, a family is chosen random. Find the probability that the family has

(i) Exactly one girl

Ans. $\frac{3}{8}$

(ii) At least one girl

Ans. $\frac{7}{8}$

(iii) At most one girl

Ans. $\frac{1}{2}$

5. Five card the ten, jack, queen, king, and ace of diamonds are well shuffled with their face downward. One card is picked up at random

(i) What is the probability that the card is the queen?

Ans. $\frac{1}{5}$

(ii) If the queen is drawn and put aside what is the probability that the second card picked up is

(a) an ace (b) a queen

Ans. $\frac{1}{4}, 0$

QUIZ

QUADRATIC EQUATION

1. What is a quadratic equation?
2. How many roots can a quadratic equation have?
3. Give the formula for finding the roots of $ax^2 + bx + c = 0$ ($a \neq 0$)
4. Give the nature of roots of the equation $ax^2 + bx + c = 0$ ($a \neq 0$)
5. Find the nature of the roots of the equation $3x^2 – 2x + 1/3 = 0$

ORAL TEST

1. A real number α is said to be a root of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c =$ _____.
2. A quadratic equation $ax^2 + bx + c = 0$ has two roots, if $b^2 – 4ac > 0$.
3. The quadratic equation $3x^2 – 4\sqrt{3}x + 4 = 0$ has two _____ roots.
4. The roots of a quadratic equation $2x^2 – 7x + 3 = 0$ are _____ and _____.
5. Two numbers whose sum is 27 and product is 182 are _____ and _____.

QUIZ

(ARITHMETIC PROGRESSIONS)

1. What is an A.P.?
2. What is meant by common difference in an A.P. ?
3. What is the formula for the nth term of an A.P.?
4. What is the formula for the sum of first n terms of an A.P. ?
5. What is the formula for the sum of first n natural numbers?

ORAL TEST

1. The common difference of a sequence of multiples of 7 is _____.
2. The difference of consecutive terms in an A.P. is always _____.
3. The sum of first 20 natural numbers is _____.
4. The sum of first eight odd natural numbers is _____.
5. The sum of first ten even natural numbers is _____.

QUIZ

(Coordinate geometry)

1. What is abscissa?
2. What is ordinate?
3. What is distance formula?
4. What is the distance of a point p(x,y) from origin?
5. Give the section formula.

ORAL TEST

1. If the area of a triangle is 0 square units, then its vertices are _____.
2. The area of a triangle whose vertices are (1 , -1), (-4, 6) and (-3, -5) is _____ square units.
3. The distance between the points (-5, 7) and (-1, 3) is _____ units.
4. _____ has been developed as an algebraic toll for studying geometry of figures.
5. The distance between the points (a,b) and (-a, -b) is _____ units.

QUIZ

(Some applications of trigonometry or heights and distance

1. Why trigonometry was invented? Give its uses.
2. What is the line of sight?
3. What is the angle of elevation?
4. What is the angle of depression?
5. What is a theodolite?

ORAL TEST

1. The other name of clinometer is _____.
2. If height of clinometer is 1 m, distance between object and clinometer is 40m and angle of elevation of object is 45^{o}, then the height of object is _____.
3. A tower stands vertically on the ground. From the point on the ground, which is 25m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60^{o} . The height of the tower is _____.

4. The angles of elevation of the top of a tower from two points at distances a and b from the base and on the same straight line with it are complementary. The height of the tower is _____.
5. A ladder 15m long just reaches the top of a vertical wall. If the ladder makes an angle of 60^o with the wall, then the height of the wall is _____.

QUIZ
(CIRCLES)

1. Define tangent to a circle.
2. How many tangent(s) is/are there at a point of circle?
3. How many tangent can be drawn to a circle from a point outside the circle?
4. Define length of a tangent.
5. What is the relation between the lengths of tangents drawn from an external point to a circle?

ORAL TEST

1. A tangent to a circle intersects it in _____ point(s).
2. A line intersecting a circle in two points is called a _____.
3. A circle can have _____ parallel tangents at the most.
4. The common point of a tangent to a circle and the circle is called _____.
5. The tangent at any point of a circle is _____ to the radius through the point of contact.

QUIZ
(Constructions)

1. What is scale factor?
2. How will you draw a tangent at a point of a circle?
3. How will you locate the centre of a circle, if it is not given?
4. How many tangents can be drawn from a point outside the circle?
5. Is it possible to draw a tangent from a point inside a circle?

ORAL TEST

1. To divide a line segment AB in the ratio m:n (m, n are positive integers), draw a ray AX so that $\angle$ BAX is an acute angle and then mark point on ray AX at equal distances such that the minimum number of these points is _____.
2. To draw a pair of tangents to a circle which are inclined to each other at an angle of 45^o, it is required to draw tangents at the end point of those two radii of the circle, the angle between which is _____.
3. To divide a line segment AB in the ration 4:5, a ray AX is drawn first such that $\angle$BAX is an acute angle and them points A1, A2, A3… are located at equal distance on the ray AX and the point B is joined to _____.
4. To construct a triangle similar to a given ΔABC with its sides 3/5 of the corresponding sides of ΔABC, first draw a ray BX such that $\angle$CBX is an acute angle and X lies on the opposite side of A with respect to BC. Tolocate points B1, B2, B3, ____ on BX at equal distances and next step is to join _____ to _____.
5. State 'True' or 'False'
    a. By geometrical construction, it is possible to divide a line segment in the ratio $3+\sqrt{5}$: $3-\sqrt{5}$.
    b. A pair of tangents can be drawn from a point P to a circle of radius 4.5 cm situated at a distance of 4 cm from the centre.
    c. By geometrical construction, it is possible to divide a line segment in the ratio $\sqrt{5}$: $1/\sqrt{5}$.
    d. A pair of tangents can be constructed to a circle inclined at an angle of 175^o.
    e. From a point P outside the circle we can draw only one tangent.
    f. We cannot locate the centre of a circle if it is not given.

QUIZ

(AREAS RELATED TO CIRCLES)

1. What is circumference of a circle? Give its formula.
2. Name the great Indian mathematician who gave an approximate value of π.
3. Give the formula for the area of a circle of radius r cm.

4. Give the formula for area of a sector of a circle having radius r and measuring an angle θ at the centre.
5. How will you find the area of a segment of a circle?

ORAL TEST

1. If the area of a circle is 154 cm^2, then its perimeter is _____.
2. Area of the largest triangle that can be inscribed in a semicircle of radius r is _____.
3. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is _____.
4. If the areas of two circles are equal, then their circumferences are _____.
5. The circles which have the same centre are called _____ circles.

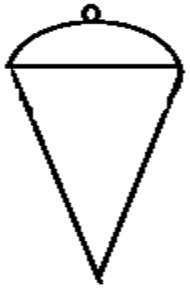
QUIZ

(SURFACE AREAS AND VOLUMES)

1. A cone of height 24cm and radius of base 6cm is made up of modeling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere.
2. A shuttle cork used for playing badminton has the shape of the combination of which basic solids?
3. What is a frustum of a right circular cone?
4. Does a frustum has two circular ends with equal radii?
5. Give the formula for the volume of the frustum of a cone.

ORAL TEST

1. A plumbline(sahul) shown in the figure is the combination of a _____ and a cone.



2. If the radii of the circular ends of a conical bucket which is 45cm high, are 28cm and 7cm then the capacity of the bucket is _____ cm^3.

3. The volume of the solid formed by joining two basic solids will actually be the _____ of the volumes of the constituents.
4. The curved surface area of the frustum of a cone is _____, where l=$\sqrt{h^2 + (r_1 - r_2)^2}$
5. If two cubes each of volumes 64cm3 are joined end to end then the surface area of the resulting cuboid is _____.

QUIZ

(PROBABILITY)

1. Define the theoretical probability of an event E.
2. What is the probability of a sure event?
3. What is an elementary event?
4. What are complementary events?
5. One card is drawn from a well shuffled deck of 52 cards. Calculate the probability that the card will be a king.

ORAL TEST

1. The probability of an impossible event is _____.
2. The probability of an event lies between _____ and _____.
3. The sum of the probabilities of all the elementary events of an experiment is _____.
4. A die is thrown once, the probability of getting a prime number is _____.
5. Two coins are tossed simultaneously. The probability of at most one tail is _____.

PUZZLES

1. Catching Fish
   If Five fishermen catch 5 fishes in 5 minutes, how long will it take fifty fishermen to catch fifty fish?

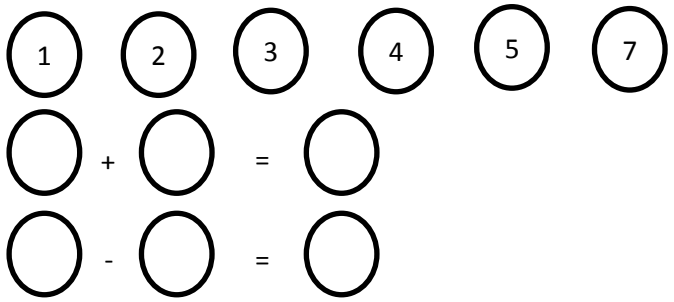
2. Look at the Division
   One day professor Agarwal went to the blackboard and demonstrated to his astonished class that one half of eight was equal to three! What did the professor do?

3. How Big
   Can you guess how big the number : ninth power nine?

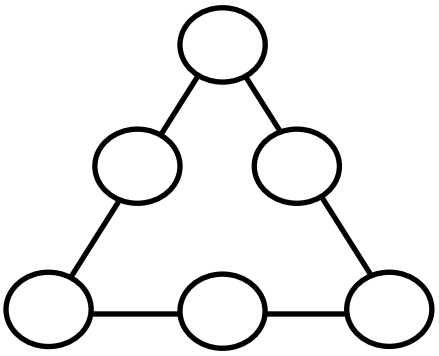
4. Counting Street Lights
   On two sides of a street, there are 35 street lights, each one is at a distance of 30 metres from the other. The street lights on one side are arranged so that each lamp fills a gap between the two other street lights on the opposite. How long is the street?

5. Who covered more distance
   Two friends Vijay and Ajay walk with constant speed of 100m/min. Vijay takes rest for 1 min after walking 100metres while Ajay takes rest for 3 min after walking 300 metres on a square path of side 400m. Both of them start from the same corner in opposite direction. Who covered more distance and when they meet?

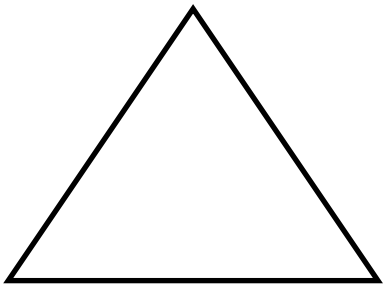
6. The missing Six
   Place the six numbers below into empty circles, so that both the equation are true. Use each number once and only once.



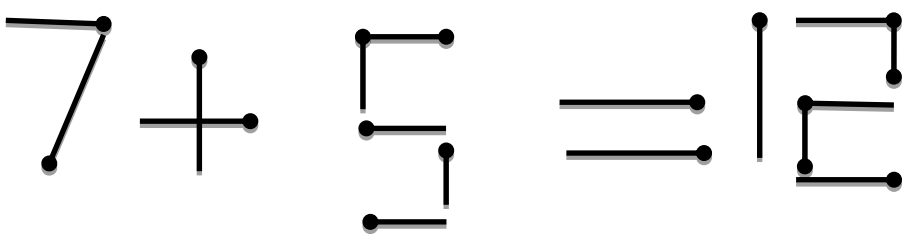
7. Magic Triangle
   Place the numbers 4 through 9 in the circles in such a way that every side of the triangle add up to 21.



8. Add up
   Here is an equilateral triangle. Add another equilateral triangle to it in such a way that you get five equilateral triangles.

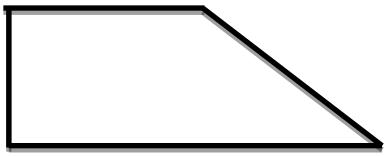


9. Magic Sticks
   Just by moving one stick, make another equation.



10. Identical Four

Divide the adjoining figure into four identical pieces.



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SAMPLE PAPER 1 SAII MATHEMATICS (CLASS X)

SECTION A

(Question numbers 1 to 4 carry 1 Mark each)

1. A tree 6 m tall casts a shadow 4 m long. At the same time, if a tower casts a shadow 10 m long then find the height of the tower ?
2. If 8^{th} term of an A.P. is 37 and 12^{th} term is 57, then find 18^{th} term.
3. How many tangents can be drawn to a circle from a point in its interior ?
4. A bag contains cards which are numbered from 2 to 90. If a card is drawn at random from the bag, then what is the probability that it bears a two digit number ?

SECTION B

(Question numbers 5 to 10 carry 2 Marks each)

5. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB+CD=AD+BC.
6. PA is a tangent to a circle of radius of 3cm with A as its point of contact. If PA=5cm find the distance of P from the centre of the circle O.
7. If the points (p, q), (m, n) and (p – q, q – n) are collinear, show that pn = qm.
8. If 5 times 5^{th} term of an AP is equal to 4 times 4^{th} term, find its ninth term.
9. Which point on y-axis is equidistant from (2 , 3) and (-4, 1).
10. Find the value of 'k' for which the quadratic equation: $(k+4)x^2 + (k+1)x + 1 = 0$ has equal roots.

SECTION C

(Question numbers 11 to 20 carry 3 Marks each)

11. Solve for 'x'

$\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$ $x \neq -1, -2, -4$

12. Find a relation between *x* and *y* such that the point P(*x*,*y*) is equidistant from the points A (2, 5) and B (-3,7).

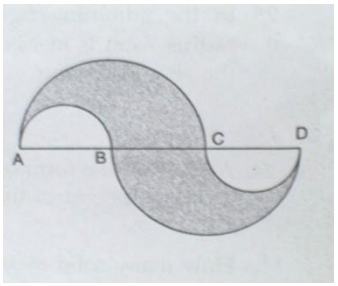
13. A toy is in the form of a cone of radius 3.5 cm surmounted on a hemisphere of the same radius. The total height of the toy is 15.5 cm. Find the surface area of the toy (take π=22/7).

14. A tree is broken by wind .The top struck the ground at an angle of 30^o at a distance of 6 meters from its root. Find the whole height of the tree(use $\sqrt{3}$=1.732)

15. In the adjoining figure semicircles are drawn taking AB, AC, BD, CD as diameters. If AB = BC = CD = 14 cm, find

i) The area of the shaded region

ii) The perimeter of the shaded region.



16. All face cards of spades are removed from a pack of 52 playing cards

and the pack is shuffled well. A card is then drawn from the remaining pack. Find the probability of getting

i) A black face card

ii) An ace or a queen

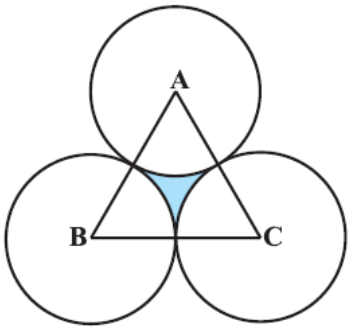
iii) Neither an ace nor a black king.

17. Find three numbers in A.P., whose sum is 21 and the product is 231.

18. How many solid metal spheres of diameter 6 cm are to be melted to form a solid cylinder?

19. A well of diameter 3m is dug 14 m deep. The earth taken out of it has been spread evenly all around it to a width of 4m to form an embankment. Find the height of the embankment.

20. The area of an equilateral triangle ABC is 17320.5 cm^2. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (Fig. 1). Find the area of the shaded region. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73205$)



SECTION D

(Question numbers 21 to 31 carry 4 Marks each)

21. Two pipes can together fill a tank in $3^{1}/_{13}$ minutes. If one pipe takes 3 minutes more than the other to fill it, find the time in which each pipe can fill the tank?

22. The 4^{th} term of the AP is equal to 3 times the first time and the 7^{th} term exceeds twice the third term by 1. Find the sum of first sixteen terms of the A.P.

23. A two digit number is such that the product of the digits is 14. When 45 is added to the number the digits are reversed. Find the number.

24. ABC is a triangle right angled at B with sides AB = 15 cm and BC = 8 cm. A circle has been inscribed in triangle ABC. Calculate the length of a diameter of the circle.

25. Construct a right triangle in which the sides other than hypotenuse are of length 5 cm and 4 cm .Then construct another triangle whose sides are $\frac{5}{3}$ times the corresponding sides of the given triangle.

26. From a window, 10 m high above the ground, of a house in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are 60^{o} and 45^{o} respectively. Find the height of the opposite house. (use $\sqrt{3}$=1.732).

27. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

28. A open bucket is in the form of a frustum of a cone whose radii of bottom and top are 7 cm and 28 cm respectively. If the capacity of the bucket is 21560 $cm^{3,}$ then find the cost of the metal sheet used in making the bucket at the rate of Rs 1 per 10 cm^{2}.

29. From a group of 2 boys and 2 girls, two children are selected at a random. What is the sample space of the experiment?

Find the probability that

i) One boy and one girl is selected

ii) At least one girl is selected

iii) At most one girl is selected

30. Find the ratio in which the point C(p,1) divides the line segment joining the points A(-4,4) and B (6,-1). Hence find the value of p.

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SAMPLE PAPER II SAII MATHEMATICS (CLASS X)

SECTION A

(Question numbers 1 to 4 carry 1 Mark each)

1. Find the 25th term of the A.P: - 5, $\frac{-5}{2}$, 0 , $\frac{5}{2}$, ...

2. A pole casts a shadow of length $2\sqrt{3}$ on the ground, when the sun's elevation is 60^{o}. Find the height of the pole

3. A game of chance consist a spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 and these are equally likely outcomes. Find the probability that the arrow will point at any factor of 8

4. Two concentric circles of radii a and b (a> b) are given. Find the length of the chord of the larger circle which touches the smaller circle.

SECTION B

(Question numbers 5 to 10 carry 2 Marks each)

5. In figure 1, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If angle TPQ= $70^{o,}$ find angle TRQ

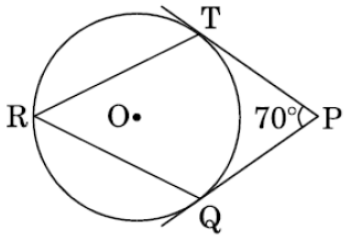

Figure 1

6. In figure 2, PQ is a chord of length 8cm of a circle of radius 5cm. The tangents at P and Q intersect at a point T. Find the lengths of TP and TQ.

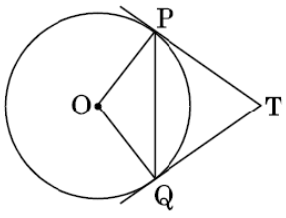

Figure 2

7. Solve for x :$x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$

8. The fourth term of an A.P is 11. The sum of the fifth and seventh terms of the A.P is 34. Find its common difference

9. Show that the points (a , a), (-a, -a) and (-$\sqrt{3}$a, $\sqrt{3}$a) are the vertices of an equilateral triangle

10. For what value of k are the points (8, 1), (3, -2k) and (k, -5) collinear?

11. Point A lies on the line segment PQ joining P(6, -6) and Q(-4, -1) in such a way that $\frac{PA}{PQ} = \frac{2}{5}$. If point P also lies on the line 3x + k (y+1) = 0. Find the value of k.

SECTION C

(Question numbers 11 to 20 carry 3 Marks each)

12. Solve for x : $x^2 + 5x - (a^2 + a - 6) = 0$

13. In an A.P, if the 12th term is -13 and the sum of its first four terms is 24. Find the sum of its first ten terms

14. A bag contains 18 balls out of which x balls are red.

i) If one ball is drawn at random from the bag. What is the probability that it is not red?

ii) If 2 more red balls are put in the bag the probability of drawing a red ball will be 9/5 times the probability of drawing a red ball in the first case. Find the value of x

15. From the top of a tower of height 50m, the angles of depression of the top and bottom of a pole are 30^o and 45^o respectively. Find

i) How far the pole is from the bottom of a tower

ii) The height of the pole (Use √3= 1.732)

16. The long and short hands of a clock are 6 cm and 4 cm long respectively. Find the sum of the distances travelled by their tips in 24 hours. (Use π = 3.14)

17. Two spheres of same metal weigh 1kg and 7kg. The radius of the smaller sphere is 3cm. The two sphere are melted to form a single big sphere. Find the diameter of the new sphere.

18. A metallic cylinder has radius 3 cm and height 5 cm. To reduce its weight, a conical hole is drilled in the cylinder. The conical hole has a radius of $\frac{3}{2}$cm and its depth $\frac{8}{9}$cm. Calculate the ratio of the volume of metal left in the cylinder to the volume of metal taken out in conical shape.

19. In figure 3, ABCD is a trapezium with AB || DC, AB = 18cm, DC = 32cm and the distance between AB and DC is 14 cm. If arcs of equal radii 7cm have been drawn with centres A, B, C and D, then find the area of the shaded region.

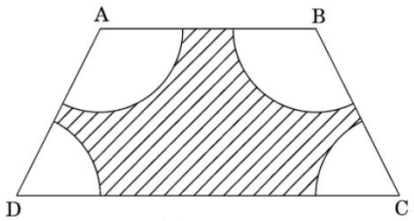

Figure 3

20. A solid right circular cone of height 60cm and radius 30cm is dropped in a right cylinder full of water of height 180cm and radius 60cm . Find the volume of water left in the cylinder, in cubic metres. (Use π = $\frac{22}{7}$)

SECTION D

(Question numbers 21 to 31 carry 4 Marks each)

21. If x = -2 is a root of the equation $3x^2 + 7x + p = 0$, find the value of k so that the roots of the equation $x^2 + k(4x + k - 1) + p = 0$ are equal.

22. Find the middle term of the sequence formed by all three digit numbers which leave a remainder 3, when divided by 4. Also find the sum of all numbers on both sides of the middle term separately

23. The total cost of a certain length of a piece of cloth is Rs.200. If the piece was 5m longer and each metre of cloth costs Rs. 2 less, the cost of the piece would have remained unchanged. How long is the piece and what is its original rate per metre.

24.Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

25. In figure 4, O is the centre of the circle and TP is the tangent to the circle from an external point T. If angle PBT= 30^o, prove that BA : AT = 2 : 1

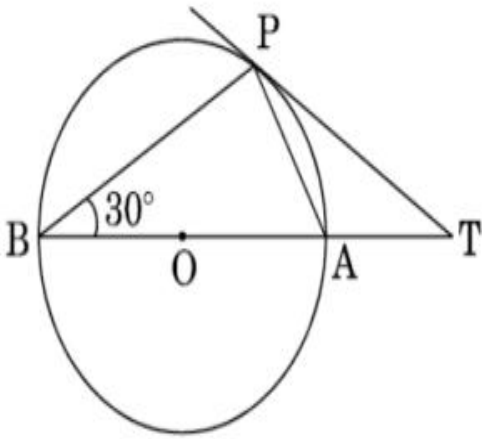

Figure 4

26. Draw a circle of radius 3cm. From a point P, 7cm away from its centre draw two tangents to the circle. Measure the length of each tangent.

27. Two poles of equal heights are standing opposite to each other on either side of the road which is 80m wide. From a point P between them on the road, the angle of elevation of the top of a pole is 60^o and the angle of depression from the top of another pole of point P is 30^o. Find the heights of the poles and the distances of the point P from the poles.

28. A box containing cards bearing numbers from 6 to 70. If one card is drawn at random from the box, find the probability that it bears

i) a one digit number

ii) a number divisible by 5

iii) an odd number less than 30

iv) a composite number between 50 and 70

29. The base BC of an equilateral triangle ABC lies on y-axis. The coordinates of point C are(0, -3). The origin is the midpoint of the base. Find the coordinates of the points A and B. Also find the coordinates of another point D such that BACD is a rhombus

30. A vessel full of water is in the form of an inverted cone of height 8cm and the radius of its top, which is open, is 5cm. 100 spherical lead balls are dropped into the vessel. One fourth of the water flows out of the vessel. Find the radius of a spherical ball.

31. Milk in a container, which is in the form of a frustum of a cone of height 30cm and the radii of whose lower and upper circular ends are 20cm and 40cm respectively, is to be distributed in a camp for flood victims. If this milk is available at the rate of Rs. 35 per litre and 880 litres of milk is needed daily for a camp, find how many such containers of milk are needed for a camp and what cost will it put on the donor agency for this. What value is indicated through by the donor agency?

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SAMPLE PAPER II SA II -Mathematics

Marking Scheme

SECTION - A

Q.No. Marks

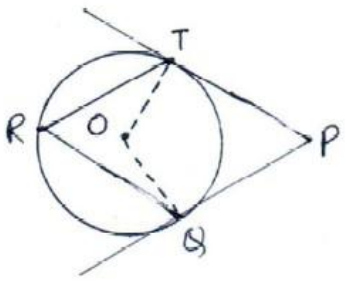
1. 55 — 1 m

2. 6 m — 1 m

3. $\frac{1}{2}$ — 1 m

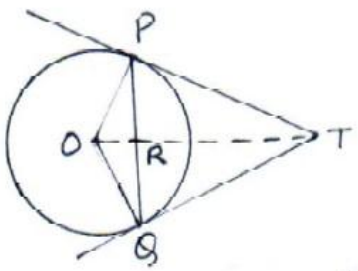
4. $2\sqrt{a^2 - b^2}$ — 1m

SECTION - B

5. 

$\angle TOQ = 180° - 70° = 110°$ — 1 m

$\Rightarrow \quad \angle TRQ = \frac{1}{2} \angle TOQ = \frac{1}{2} \times 110° = 55°$ — 1 m

6. 

$OR = \sqrt{OP^2 - PR^2} = \sqrt{25 - 16} = 3cm$

Let RT be x

$PT^2 = PR^2 + RT^2 = 16 + x^2$ — ½ m

Also $PT^2 = OT^2 - OP^2 = (3 + x)^2 - 25$

$= x^2 + 6x - 16$

$\Rightarrow \quad 16 + x^2 = x^2 + 6x - 16$ — 1 m

$\Rightarrow \quad x = \frac{16}{3}$

Thus $TP = TQ = \sqrt{16 + \frac{256}{9}} = \frac{20}{3}$ cm ½ m

7. $x^2 - \sqrt{3}\,x - x + \sqrt{3} = 0$ ½ m

$\Rightarrow \quad (x - \sqrt{3})(x - 1) = 0$ 1 m

$\Rightarrow \quad x = \sqrt{3}, 1$ ½ m

8. Let the first term be a and the common difference be d

$a + 3d = 11$ ..... (i) ½ m

$(a + 4d) + (a + 6d) = 34$

$\Rightarrow \quad a + 5d = 17$ ..... (ii) 1 m

Solving (i) & (ii)

$a = 2, \ d = 3$ ½ m

9. $AB = \sqrt{(a + a)^2 + (a + a)^2} = 2\sqrt{2}\,a$ ½ m

$BC = \sqrt{(-a + \sqrt{3}a)^2 + (-a - \sqrt{3}a)^2} = 2\sqrt{2}\,a$ ½ m

$AC = \sqrt{(a + \sqrt{3}a)^2 + (a - \sqrt{3}a)^2} = 2\sqrt{2}\,a$ ½ m

Since AB = BC = AC, therefore ABC is an equilateral triangle ½ m

10. The given points (8, 1) (3, – 2k) and (k, – 5) are collinear

$\Rightarrow$ Area of the triangle formed = 0

$\Rightarrow \quad \frac{1}{2}[\,8(-2k + 5) + 3(-5 - 1) + k(1 + 2k)\,] = 0$ 1 m

$\Rightarrow \quad 2k^2 - 15k + 22 = 0$ ½ m

$\Rightarrow \quad (k-2)(2k-11) = 0$

$\Rightarrow \quad k = 2, \ \frac{11}{2}$ ½ m

SECTION - C

11. Point $P(6, -6)$ lies on the line $3x + k(y+1) = 0$

$\Rightarrow \quad 18 + k(-6+1) = 0$ 1½ m

$\Rightarrow \quad k = 18/5$ 1½ m

12. $x^2 + 5x - (a^2 + a - 6) = 0$

$\therefore \quad x = \frac{-5 \pm \sqrt{25 + 4(a^2 + a - 6)}}{2}$ 1 m

$= \frac{-5 \pm (2a+1)}{2}$ 1 m

$= \frac{2a-4}{2}, \ \frac{-2a-6}{2}$

$\therefore \quad x = a - 2, \ -a - 3$ ½+½ m

13. $a + 11d = -13$ ..... (i) ½ m

$S_4 = 2(2a + 3d) = 24$

$\Rightarrow \quad 2a + 3d = 12$ ..... (ii) 1 m

Solving (i) and (ii)

$a = 9, \ d = -2$ 1 m

Thus $S_{10} = 5[18 - 18] = 0$ ½ m

14. (i) P (ball not red) $= 1 - \frac{x}{18}$ or $\frac{18 - x}{18}$ 1 m

(ii) Total number of balls = 20, red balls = x + 2

P (Red ball) $= \frac{x + 2}{20}$ ½ m

According to the question $\frac{x + 2}{20} = \frac{9}{8} \times \frac{x}{8}$ 1 m

$\Rightarrow \quad x = 8$ ½ m

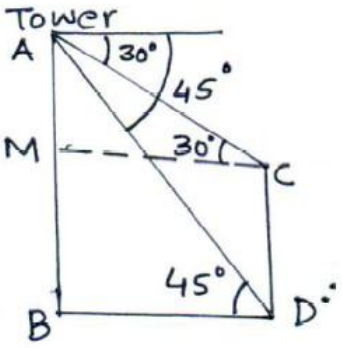
15. 

figure ½ m

AB = 50 m

$\tan 45° = \frac{AB}{BD} = 1$

$\Rightarrow$ AB = BD = 50 m. 1 m

Distance of pole from bottom of tower = 50 m

$\tan 30° = \frac{AM}{MC} = \frac{AM}{BD}$

$\Rightarrow \quad AM = \frac{50}{\sqrt{3}}$ or 28.87 m. 1 m

Height of pole $= CD = BM = 50 - \frac{50}{\sqrt{3}}$

$= 21.13$ m ½ m

16. Long hand makes 24 rounds in 24 hours

Short hand makes 2 rounds in 24 hours } 1 m

Distance traveled by long hand in 24 rounds = $24 \times 12\ \pi$

$= 288\ \pi$ cm. ½ m

Distance traveled by short hand in 2 rounds = $2 \times 8\,\pi$

$= 16\,\pi$ cm. ½ m

Sum of the distance = $288\,\pi + 16\,\pi = 304\,\pi$

$= 304 \times 3.14$

$= 954.56$ cm. 1 m

17. Volume of small sphere = $\frac{4}{3}\pi\,(3)^3 = 36\pi\ \text{cm}^3$ ½ m

Volume of big sphere = $7 \times 36\,\pi = 252\pi\ \text{cm}^3$ ½ m

Volume of sphere formed = $36\,\pi + 252\,\pi = 288\,\pi\ \text{cm}^3$

$\therefore \quad \frac{4}{3}\pi\,r^3 = 288\,\pi$ 1 m

$\Rightarrow \quad r = 6$ cm. ½ m

Diameter of the sphere = 12 cm. ½ m

18. Volume of the cylinder = $\pi\,(3)^2 \times 5 = 45\,\pi\ \text{cm}^3$ ½ m

Volume of conical hole = $\frac{1}{3}\pi\left(\frac{3}{2}\right)^2 \times \frac{8}{9} = \frac{2}{3}\pi\ \text{cm}^3$ 1 m

Metal left in the cylinder = $\left(45\,\pi - \frac{2\,\pi}{3}\right) = \frac{133\,\pi}{3}\ \text{cm}^3$ 1 m

Required ratio is $\frac{133}{3}\pi : \frac{2}{3}\pi = 133 : 2$ ½ m

19. Area of trapezium = $\frac{1}{2}(18 + 32) \times 14 = 350\ \text{cm}^2$ ½ m

Area of four arcs = $\pi\,(7)^2 = 154\ \text{cm}^2$ 1½ m

Area of shaded region = $350 - 154 = 196\ \text{cm}^2$ 1 m

20. Volume of water in cylinder = $\pi (60)^2 \times 180 = 648000\ \pi\ cm^3$. 1 m

Volume of solid cone = $\frac{1}{3}\pi (30)^2 \times 60 = 1800\ \pi\ cm^3$. 1 m

Volume of water left in cylinder = $648000\ \pi - 1800\ \pi$

$= 630000\ \pi\ cm^3$.

$= 1.98\ m^3$. 1 m

SECTION - D

21. $x = -2$ is root of the equation $3x^2 + 7x + p = 0$

$\Rightarrow\ 3(-2)^2 + 7(-2) + p = 0$

$\Rightarrow\ p = 2$ 1 m

Roots of the equation $x^2 + 4k\ x + k^2 - k + 2 = 0$ are equal

$\Rightarrow\ 16k^2 - 4(k^2 - k + 2) = 0$ 1 m

$\Rightarrow\ 3k^2 + k - 2 = 0$

$\Rightarrow\ (3k - 2)(k + 1) = 0$ 1 m

$\Rightarrow\ k = \frac{2}{3}, -1$ 1 m

22. The three digit number which leave remainder 3

when divided by 4 are

103, 107, 111, ..., 999 1 m

$\therefore\ 999 = 103 + (n - 1)\ 4$

$\Rightarrow\ n = 225$ ½ m

Therefore $\frac{225 + 1}{2} = 113$th term is middle term ½ m

Middle term = $103 + 112 \times 4 = 551$ 1 m

Sum of first 112 terms $= \frac{112}{2}(206 + 111 \times 4) = 36400$ ½ m

Sum of last 112 terms $= \frac{112}{2}(1110 + 111 \times 4) = 87024$ ½ m

23. Let lenght of cloth be x m.

Cost per meter = Rs. $\frac{200}{x}$

New lenght of cloth = (x + 5) m.

New cost per meter = Rs. $\left(\frac{200}{x} - 2\right)$ 1 m

$\therefore \quad (x + 5)\left(\frac{200}{x} - 2\right) = 200$ 1 m

$\Rightarrow \quad x^2 + 5x - 500 = 0$

$\Rightarrow \quad (x + 25)\ (x - 20) = 0$

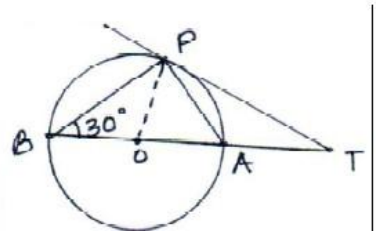
$\Rightarrow \quad x = 20, \quad x \neq -25$ 1 m

Length of piece = 20 m ½ m

Original cost per meter = Rs. $\frac{200}{20}$ = Rs. 10 ½ m

24. Correct figure given, to prove and construction ½ x 4 =2 m

Correct proof 2 m

25. 

$\angle AOP = 2 \times 30° = 60°$ ½ m

$\angle OAP = 180° - 30° - 90° = 60°$

$\therefore \quad OP = PA$ 1 m

Also $\angle ATP = \angle APT = 30°$ ½ m

$\therefore$ $AP = AT = OP = OA$ 1 m

Hence $BA = 2\,OA = 2\,AT$

$\Rightarrow$ $BA : AT = 2 : 1$ 1 m

26. Correct construction 3 m

Measure of each tangent = 6.3 cm (approx) 1 m

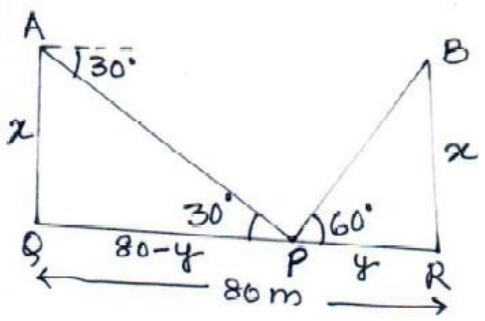
27. 

Figure 1 m

$\tan 60° = \frac{x}{y}$

$\Rightarrow$ $x = y\sqrt{3}$ ..... (i) ½ m

$\tan 30° = \frac{x}{80 - y}$

$\Rightarrow$ $\sqrt{3}\,x = 80 - y$ ..... (ii) 1 m

Solving (i) and (ii)

$y = 20$, $x = 20\sqrt{3}$ m. ½ m

Height of pole = $20\sqrt{3}$ m.

PR = 20 m.

OP = 80 – 20 = 60 m. 1 m

28. Total number of cords = 65

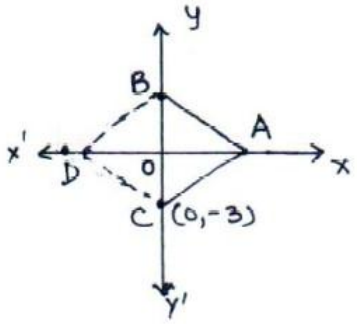
(i) P (a one digit number) = 4/65 1 m

(ii) P (a number divisible by 5) = $\frac{13}{65} = \frac{1}{5}$ 1 m

(iii) P (an odd number less than 30) = $\frac{12}{65}$ 1 m

(iv) P (a composite number between 50 and 70) $= \frac{15}{65} = \frac{3}{13}$ 1 m

29.



Coordinates of point B are (0, 3) ½ m

$\therefore$ BC = 6 units

Let coordinates of point A be (x, 0) ½ m

$\Rightarrow$ $AB = \sqrt{x^2 + 9}$

$\because$ AB = BC

$\therefore$ $x^2 + 9 = 36$ 1 m

$\Rightarrow$ $x^2 = 27$ $\Rightarrow$ $x = \pm 3\sqrt{3}$

Coordinates of point A = $\left(3\sqrt{3}, 0\right)$ 1 m

Since BACD is a thombus $\Rightarrow$ AB = AC = CD = DB

$\therefore$ Coordinates of point D = $\left(-3\sqrt{3}, 0\right)$ 1 m

30. Volume of water in cone $= \frac{1}{3}\pi\left(5^2\right) \times 8 = \frac{200\,\pi}{3}\ \text{cm}^3$ ½ m

Volume of water flows out $= \frac{1}{4}\left(\frac{200\,\pi}{3}\right) = \frac{50\,\pi}{3}\ \text{cm}^3$ 1 m

Let radius of one spherical ball be x cm.

$\therefore$ $\frac{4}{3}\pi\left(x^3\right) \times 100 = \frac{50\,\pi}{3}$ 1½ m

$\Rightarrow$ $x^3 = \frac{1}{8}$

$\Rightarrow$ $x = \frac{1}{2}$ cm or 0.5 cm. 1 m

31. Volume of milk in a container $= \frac{\pi\, 30}{3}(1600 + 400 + 800)$ 1 m

$= 88000\ cm^3$

$= 88$ litres 1 m

Number of containers needed $= \frac{880}{88} = 10$ ½ m

Cost of milk = Rs. $88 \times 10 \times 35$

= Rs. 30800 ½ m

Value 1 m

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